

1 Total force from the electromagnetic field of the source

1.1 Electric force

The electric force on a dipolar particle of polarizability α_e is [?]

$$F_i^e(\mathbf{r}) = \frac{\varepsilon_0 \varepsilon_1}{2} \Re \{ \langle \alpha_e E_j^*(\mathbf{r}) \partial_i E_j(\mathbf{r}) \rangle \}. \quad (1)$$

The electric field from the source is [2]

$$E_i^{inc}(\mathbf{r}) = \mu_0 \mu_2 \omega^2 \int_V G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega) P_j(\mathbf{r}', \omega) d^3 \mathbf{r}'. \quad (2)$$

where $G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega)$ is the electric Green function from the polarization currents and contains the transmission Fresnel coefficients ($t_{s,p}$). $G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega)$ is expressed on using the angular spectrum of plane waves [2, 3]

$$G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{2} \int \frac{d^2 \mathbf{K}}{(2\pi)^2} G_{ij}^{E,P}(\mathbf{K}) e^{i\mathbf{K}(\mathbf{R}-\mathbf{R}')} e^{i\gamma_1 z - i\gamma_2 z'}. \quad (3)$$

where $G_{ij}^{E,P}(\mathbf{K}) = \frac{1}{\gamma_2} (\hat{s}_i t_{21}^s \hat{s}_j + \hat{p}_{1i}^+ t_{21}^p \hat{p}_{2j}^+)$, $\hat{\mathbf{s}} = \hat{\mathbf{K}} \times \hat{\mathbf{z}}$, $\hat{\mathbf{p}}_i^\pm = -[\gamma_i \hat{\mathbf{K}} \mp K \hat{\mathbf{z}}] / (n_i k_0)$, $\gamma_i = \sqrt{\varepsilon_i \mu_i k_0^2 - K^2}$ and $n_i = \sqrt{\varepsilon_i \mu_i}$. n_1 (n_2) is the refractive index of the medium placed at $z < 0$ ($z > 0$).

The self-correlation function $\langle E_j^*(\mathbf{r}) E_j(\mathbf{r}) \rangle$ at the position of the particle, i.e., at $\mathbf{r} = \mathbf{r}_0$ will be

$$\langle E_j^*(\mathbf{r}_0) E_j(\mathbf{r}_0) \rangle = \mu_0^2 \mu_2^2 \omega^4 \int_{V_1, V_2} G_{ij}^{E,P*}(\mathbf{r}, \mathbf{r}', \omega) G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega) W_{kl}^{(P)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) d^3 \mathbf{r}'_1 d^3 \mathbf{r}'_2, \quad (4)$$

$W_{kl}^{(P)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \langle P_k^*(\mathbf{r}'_1) P_l(\mathbf{r}'_2) \rangle$ being the cross-spectral density tensor of the source polarization. Substituting $W_{kl}^{(P)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ into Eq. (4) we have $\langle E_j^*(\mathbf{r}) E_j(\mathbf{r}) \rangle$ explicitly expressed in terms of the angular spectrum

$$\begin{aligned} \langle E_j^*(\mathbf{r}_0) E_j(\mathbf{r}_0) \rangle &= \frac{\mu_0^2 \mu_2^2 \omega^4}{4} \int_{V_1, V_2} \left[\int_{\mathbf{K}_1, \mathbf{K}_2} \frac{d^2 \mathbf{K}_1}{(2\pi)^2} \frac{d^2 \mathbf{K}_2}{(2\pi)^2} G_{ij}^{E,P*}(\mathbf{r}, \mathbf{r}', \omega) G_{ij}^{E,P}(\mathbf{r}, \mathbf{r}', \omega) e^{-i\mathbf{K}_1(\mathbf{R}-\mathbf{R}'_1)} e^{i\mathbf{K}_2(\mathbf{R}-\mathbf{R}'_2)} \right. \\ &\times \left. e^{-(i\gamma_{1,1}^* z - i\gamma_{2,1}^* z')} e^{i\gamma_{1,2} z - i\gamma_{2,2} z'} \right] W_{kl}^{(P)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) d^3 \mathbf{r}'_1 d^3 \mathbf{r}'_2. \end{aligned} \quad (5)$$

We shall assume the correlation with a Gaussian profile [4]

$$W_{ij}^{(P)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \mathcal{S}^{(P)}(\omega) \exp(-(|\mathbf{r}'_1 - \mathbf{r}'_2|)^2 / 2\sigma^2) \delta_{ij} / (2\pi)^{3/2} \sigma^3, \quad (6)$$

where $\mathcal{S}^{(P)}(\omega)$ is the normalized spectrum of the source and σ represents its correlation length of the source. With a suitable change of variables, the integration over the lateral space coordinates gives a two-dimensional delta function $\delta^{(2)}[\mathbf{K}_1 - \mathbf{K}_2]$ which will simplify one of the integrations over \mathbf{K} . The rest of integrals lead to a more simplified equation:

$$\begin{aligned} \langle E_j^*(\mathbf{r}_0) E_j(\mathbf{r}_0) \rangle &= \frac{\mu_0^2 \mu_2^2 \omega^4}{4} \frac{1}{(2\pi)^{6/5}} \mathcal{S}^{(P)}(\omega) \\ &\times \int_{\mathbf{K}} \frac{1}{|\gamma_2|^2} e^{-\frac{(\mathbf{K}\sigma)^2}{2}} \left[|t_{21}^s|^2 + |t_{21}^p|^2 \frac{1}{n_1^2 k_0^2} (|\gamma_1|^2 + K^2) \frac{1}{n_2^2 k_0^2} (|\gamma_2|^2 + K^2) \right] \\ &\times e^{-2z_0 \Im \gamma_1} \frac{1}{2 \Im \gamma_2} e^{-\frac{1}{2} \sigma^2 \Re \gamma_2^2} d^2 \mathbf{K}. \end{aligned} \quad (7)$$

Next, we will divide the force into conservative and non-conservative components.

1.1.1 Conservative force.

The conservative electric force $F_i^{e,cons} = \text{Re} \alpha \partial_i \langle E_j^*(\mathbf{r}) E_j(\mathbf{r}) \rangle / 4$ due to the electric field is given by

$$\begin{aligned} F_z^{e-cons} &= -\frac{\varepsilon_0 \varepsilon_1}{4} \Re \{ \alpha_e \} \frac{\mu_0^2 \mu_2^2 \omega^4}{4} \frac{1}{(2\pi)^{1/5}} \mathcal{S}^{(P)}(\omega) \\ &\times \int_{K=k_0}^{K=+\infty} 2\sqrt{K^2 - k_0^2} \frac{1}{|\gamma_2|^2} e^{-\frac{(\mathbf{K}\sigma)^2}{2}} \left[|t_{21}^s|^2 + |t_{21}^p|^2 \frac{1}{n_1^2 k_0^2} (|\gamma_1|^2 + K^2) \frac{1}{n_2^2 k_0^2} (|\gamma_2|^2 + K^2) \right] \\ &\times e^{-2z_0 \Im \gamma_1} \frac{1}{2\Im \gamma_2} e^{-\frac{1}{2}\sigma^2 \Re \gamma_2^2} K dK, \end{aligned} \quad (8)$$

where only the third Cartesian component and the contribution is solely due to the evanescent modes. This integration is numerically resolved.

1.1.2 Non conservative force.

The non-conservative force $F_i^{e,nc} = \text{Im} \alpha \text{Im} \langle E_j^*(\mathbf{r}) \partial_i E_j(\mathbf{r}) \rangle / 2$ due to the electric field, is determined in a similar way, now calculating $\langle E_j^*(\mathbf{r}) \partial_i E_j(\mathbf{r}) \rangle$ instead of $\langle E_j^*(\mathbf{r}) E_j(\mathbf{r}) \rangle$. Hence,

$$\begin{aligned} F_i^{e-nc} &= \frac{\varepsilon_0 \varepsilon_1}{2} \text{Im} \alpha_e \frac{\mu_0^2 \mu_2^2 \omega^4}{4} \frac{1}{(2\pi)^{1/5}} \mathcal{S}^{(P)}(\omega) \\ &\times \int_{K=0}^{K=k_0} \sqrt{k_0^2 - K^2} \frac{1}{|\gamma_2|^2} e^{-\frac{(\mathbf{K}\sigma)^2}{2}} \left[|t_{21}^s|^2 + |t_{21}^p|^2 \frac{1}{n_1^2 k_0^2} (|\gamma_1|^2 + K^2) \frac{1}{n_2^2 k_0^2} (|\gamma_2|^2 + K^2) \right] \\ &\times \frac{1}{2\Im \gamma_2} e^{-\frac{1}{2}\sigma^2 \Re \gamma_2^2} K dK, \end{aligned} \quad (9)$$

where only the homogeneous waves give a non-zero value which is constant for any value of \mathbf{r} . This integration is resolved numerically.

1.2 Magnetic force

The magnetic force for a magnetodielectric particle is [1]

$$F_i^m(\mathbf{r}) = \frac{\mu_0 \mu_1}{2} \Re \{ \langle \alpha_m H_j^*(\mathbf{r}) \partial_i H_j(\mathbf{r}) \rangle \}. \quad (10)$$

The magnetic field emitted by the source is Maxwell's equations,

$$H_j^{inc}(\mathbf{r}) = -i\omega \int_V G_{jk}^{H,P}(\mathbf{r}, \mathbf{r}', \omega) P_k(\mathbf{r}', \omega) d^3 \mathbf{r}', \quad (11)$$

where the magnetic Green's function is

$$G_{jk}^{H,P}(\mathbf{r}, \mathbf{r}', \omega) = \frac{k_0 n_2}{2} \int \frac{d^2 \mathbf{K}}{(2\pi)^2} G_{km}^{H,P}(\mathbf{K}) e^{i\mathbf{K}(\mathbf{R}-\mathbf{R}')} e^{i\gamma_1 z - i\gamma_2 z'}, \quad (12)$$

and $G_{kl}^H(\mathbf{K}) = \frac{1}{\gamma_2} (\hat{p}_{1k}^+ t_{21}^s \hat{s}_l - \hat{s}_k t_{21}^p \hat{p}_{2l}^+)$. The rest of the calculation is similar to the one described in Section 1.1.

1.3 Interaction force

The force from the interference between the electric and magnetic dipoles is [1]

$$F_i^{e-m}(\mathbf{r}) = -\varepsilon_0 \varepsilon_1 \frac{Z k_0^4}{12\pi} \Re \{ (\alpha_e^* \alpha_m) \langle \mathbf{E}^* \times \mathbf{H} \rangle_i \}, \quad (13)$$

where $Z = \sqrt{\mu_0 \mu_1 / (\varepsilon_0 \varepsilon_1)}$. Once we have defined the electric and magnetic fields emitted by the source [cf. Eqs. (2-3) and (11-12)], one can calculate the cross product $\langle \mathbf{E}^* \times \mathbf{H} \rangle$,

2 Total force from the electromagnetic field emitted by the particle induced electric and magnetic dipoles

2.1 Electric force

The field emitted by the electric (magnetic) dipole p (m), after reflections on the source surface $z = 0$ is

$$\begin{aligned} E_i^p(\mathbf{r}) &= \mu_0 \mu_2 \omega^2 \int_V G_{ij}^{E,p}(\mathbf{r}, \mathbf{r}', \omega) p_j(\mathbf{r}', \omega) \delta(\mathbf{r}' - \mathbf{r}_0) d^3 \mathbf{r}', \\ &= \mu_0 \mu_2 \omega^2 G_{ij}^{E,p}(\mathbf{r}, \mathbf{r}_0, \omega) p_j(\mathbf{r}_0, \omega), \end{aligned} \quad (14)$$

$$E_i^m(\mathbf{r}) = \frac{Z_0 i \omega}{c} G_{ij}^{H,m \leftrightarrow}(\mathbf{r}, \mathbf{r}', \omega) m_j(\omega), \quad (15)$$

where the electric Green's function contains the properties of the source through the reflection Fresnel coefficients ($r_{s,p}$)

$$G_{ij}^{E,p}(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{2} \int \frac{d^2 \mathbf{K}}{(2\pi)^2} G_{ij}^{E,p}(\mathbf{K}) e^{i\mathbf{K}(\mathbf{R}-\mathbf{R}')} e^{i\gamma_1(z+z')}, \quad (16)$$

$$G_{ij}^{H,m \leftrightarrow}(\mathbf{r}, \mathbf{r}', \omega) = \frac{k_0 n_2}{2} \int \frac{d^2 \mathbf{K}}{(2\pi)^2} G_{ij}^{H,m \leftrightarrow}(\mathbf{K}) e^{i\mathbf{K}(\mathbf{R}-\mathbf{R}')} e^{i\gamma_1(z+z')}. \quad (17)$$

$G_{ij}^{E,p}(\mathbf{K}) = \frac{1}{\gamma_1} (\hat{s}_i r_{12}^s \hat{s}_j + \hat{p}_{1i}^+ r_{12}^p \hat{p}_{1j}^-)$ and $G_{ij}^{H,m \leftrightarrow}(\mathbf{K}) = \frac{1}{\gamma_1} (\hat{p}_{1i}^+ r_{12}^p \hat{s}_j - \hat{s}_i r_{12}^s \hat{p}_{1j}^-)$. The superscript \leftrightarrow denotes that the electric field generated by the magnetic dipole has the same Green's function as the magnetic field radiated by the magnetic dipole with the interchange $r_s \leftrightarrow r_p$. Notice also that in the Green function described above there is no free-space term. This is due to the multiple scattering of the dipole field with the source surface, (see for example [5, 6]).

The correlation function $\langle E_i^*(\mathbf{r}) E_i(\mathbf{r}) \rangle$ at the position of the particle is obtained on considering that the cross-correlation between the electric and magnetic dipoles is zero, i.e., $\langle p_i^* m_j \rangle = 0$, therefore

$$\langle E_i^*(\mathbf{r}_0) E_i(\mathbf{r}_0) \rangle = \langle (E_i^{p*}(\mathbf{r}_0) + E_i^{m*}(\mathbf{r}_0)) (E_i^p(\mathbf{r}_0) + E_i^m(\mathbf{r}_0)) \rangle = \langle E_i^{p*}(\mathbf{r}_0) E_i^p(\mathbf{r}_0) \rangle + \langle E_i^{m*}(\mathbf{r}_0) E_i^m(\mathbf{r}_0) \rangle, \quad (18)$$

each ensemble average being

$$\langle E_j^{p*}(\mathbf{r}_0) E_j^p(\mathbf{r}_0) \rangle = \mu_0^2 \mu_1^2 \omega^4 G_{jk}^{E,p*}(\mathbf{r}_0, \mathbf{r}_0, \omega) G_{jl}^{E,p}(\mathbf{r}, \mathbf{r}', \omega) \langle p_k^*(\mathbf{r}_0, \omega) p_l(\mathbf{r}_0, \omega) \rangle, \quad (19)$$

$$\langle E_j^{m*}(\mathbf{r}_0) E_j^m(\mathbf{r}_0) \rangle = \left(\frac{Z_0 \omega}{c} \right)^2 G_{jk}^{H,m \leftrightarrow*}(\mathbf{r}_0, \mathbf{r}_0, \omega) G_{jl}^{H,m \leftrightarrow}(\mathbf{r}, \mathbf{r}', \omega) \langle m_k^*(\mathbf{r}_0, \omega) m_l(\mathbf{r}_0, \omega) \rangle, \quad (20)$$

and

$$p_k(\mathbf{r}, \omega) = \varepsilon_0 \varepsilon_1 \alpha_e(\omega) E_k^{inc}(\mathbf{r}_0, \mathbf{r}_1, \omega), \quad (21)$$

$$p_k(\mathbf{r}, \omega) = \alpha_m(\omega) H_k^{inc}(\mathbf{r}_0, \mathbf{r}_1, \omega). \quad (22)$$

The correlation tensor $\langle E_k^{inc*}(\mathbf{r}_0, \mathbf{r}_1, \omega) E_l^{inc}(\mathbf{r}_0, \mathbf{r}_1, \omega) \rangle$ has been calculated in the Section 1.1, where \mathbf{r}_1 is a point of the half-space occupied by source. After a somewhat protracted calculation one obtains that at the particle position one has that $\langle E_j^{m*}(\mathbf{r}_0) E_j^m(\mathbf{r}_0) \rangle = 0$, thus the only term remaining different from zero is $\langle E_j^{p*}(\mathbf{r}_0) E_j^p(\mathbf{r}_0) \rangle$:

$$\begin{aligned} \langle E_j^{p*}(\mathbf{r}_0) E_j^p(\mathbf{r}_0) \rangle &= \mu_0^2 \mu_1^2 \omega^4 |\varepsilon_0|^2 |\varepsilon_1|^2 |\alpha_e|^2 G_{jk}^{E,p*}(\mathbf{r}_0, \mathbf{r}_0, \omega) G_{jl}^{E,p}(\mathbf{r}_0, \mathbf{r}_0, \omega) \langle E_k^{inc*}(\mathbf{r}_0, \mathbf{r}_1, \omega) E_l^{inc}(\mathbf{r}_0, \mathbf{r}_1, \omega) \rangle \\ &= \mu_0^2 \mu_1^2 \omega^4 |\varepsilon_0|^2 |\varepsilon_1|^2 |\alpha_e|^2 \frac{\mu_0^2 \omega^2}{4} \frac{1}{(2\pi)^{1/5}} \mathcal{S}^{(P)}(\omega) \end{aligned}$$

$$\begin{aligned}
& \times \left| \frac{i}{2} \int \frac{2\pi K dK}{(2\pi)^2} \frac{1}{\gamma_1} \left(r_{12}^s + \frac{r_{12}^p}{(n_1 k_0)^2} (\gamma_1^2 - K^2) \right) e^{i\gamma_1(z_0 + z'_1)} \right|^2 \\
& \times \int_K \frac{2\pi}{|\gamma_2|^2} e^{-\frac{(\kappa_\sigma)^2}{2}} \left(|t_{21}^s|^2 + |t_{21}^p|^2 \frac{1}{n_1^2 n_2^2 k_0^4} (|\gamma_1|^2 + K^2) (|\gamma_2|^2 + K^2) \right) e^{-2z_0 \Im \gamma_1} \frac{1}{2\Im \gamma_2} e^{-\frac{1}{2}\sigma^2 \Re \gamma_2^2} K dK.
\end{aligned} \tag{23}$$

After completing the whole procedure, we derive the conservative plus the non-conservative parts of the total electric force.

2.2 Magnetic force

The magnetic field emerged from either the electric and the magnetic dipoles is calculated by using the following substitutions [7]:

$$E_i^m(\mathbf{r}) = -\frac{Z_0}{c} H_i^p(\mathbf{r}), \tag{24}$$

$$H_i^m(\mathbf{r}) = \frac{1}{Z_0 c} E_i^p(\mathbf{r}), \tag{25}$$

$$\mathbf{p} \rightarrow \mathbf{m}, \tag{26}$$

$$r^s \leftrightarrow r^p, \tag{27}$$

The procedure to obtain the force follows the same steps as described in Section 2.1

2.3 Interaction force

Once we have characterized the electric and magnetic fields emerging from the electric and magnetic dipoles [cf. Eqs. (14-17)], one calculates the cross product $\langle \mathbf{E}^* \times \mathbf{H} \rangle$ in a way similar to that of see section 2.1.

References

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